International Journal of Theoretical Physics, Vol. 11, No. 6 (1974), pp. 387-394

Zero Value of the Schwarzschild Mass for an Asymptotically Euclidian System of Gravitational Waves

V. N. FOLOMESHKIN

Institute of High Energy Physics, Serpukhov, Moscow District, USSR

Received: 10 *June* 1974

Abstract

A class of the asymptotically Euclidian space-times is shown to exist for which the Schwarzschild mass is equal to zero. The coordinate atlases of these space-times satisfy two additional conditions: $\partial_k(-gg^{0k}) = 0$ and $\Gamma^0_{ik}\,\partial_0 g^{ik} - \Gamma^k_{ik}\,\partial_0 g^{0i} = 0$. In a T-orthogonal metric $ds^2 = g_{00}\,dt^2 - g_{\alpha\beta}\,dx^\alpha\,dx^\beta$ these conditions take a simple form: $\partial_0\,(\det g_{\alpha\beta}) = 0$ and $(\partial_0 g^{\alpha\beta})(\partial_0 g_{\alpha\beta}) = 0$.

> Couldn't it be that the energy-momentum are not just quite conserved quantities in General Relativity.
>
> (Penrose, 1968)

1. *Introduction*

In most papers on General Relativity in which the problem of energy-momentum was considered (Lorentz, 1915, 1916; Gilbert, 1915, 1924; Einstein, 1916; Klein, 1917, 1918; Weyl, 1917, 1918; and then the innumerable repetition in the papers of Bessel-Hagen (1921), Eddington (1923), Tolman (1930, 1934), Born (1937), Pauli (1941), Iskraut (1942), de Wet (1947), Schrödinger (1948, 1951), Bergmann *et al.* (1948–1956), Hill (1951), Goldberg (1953, 1958), Trautman (1956, 1964), Møller (1958, 1959, 1961), Mitskevich (1958), Fletcher (1960)) the 'energy' was *defined* with the generalised 'energy-momentum' complex ('pseudo-tensor'). It is known that the integral quantities obtained with the generalised complex have no definite transformation properties. The generalised complex is a generalisation for the case of a continuous medium of the notion of the generalised momentum which, in the common case, has no relation to the usual energy-momentum vector. The physical sense of the generalised complex depends on the choice of a particular coordinate atlas. Therefore the appearance of the known Lorentz (1916c, d) paradox, considered also by Schrödinger (1918), Bauer (1918) and Møller (1964), is not astonishing. It is not amusing when one author obtains negative values for the gravitational wave 'energy' (Hu, 1947; Peres, 1959; Havas & Goldberg, 1962; Sexl, 1966; Petrov, Piragas & Dobrovolsky, 1968), other authors obtain the zero value (Brdicka, 1951; Infeld, 1953, 1956, 1959; Scheidegger, 1951, 1953, 1955; Møller, 1958b; Trautman, 1958; Pirani, 1961; Misner, 1963; Cahen & Sengier-Diels, 1963; Langer, 1963; Kuchar & Langer, 1963; Shirokov, 1972; Folomeshkin, 1970), and many others obtain a positive value.

Folomeshkin (1967, 1969, 1971, 1972), Ray (1968) and Plybon (1971) considered the covariant formulation of the differential conservation laws in Riemannian space. (As Pauli (1921) noted, 'in the general case and in the principal problems of a theory the general covariance is necessary'.) It follows from this formulation that the canonical energy-momentum tensor of gravitational waves is equal to zero (Folomeshkin, 1967, 1969, 1971). Therefore it seems possible that in some cases the Schwarzschild mass (S-mass) of an asymptotically Euclidian system of gravitational waves can be equal to zero.

Brill (1959), Araki (1959), Arnowitt, Deser & Misner (1960), Brill, Deser & Faddeev (1968) and Brill & Deser (1968) have shown that the S-mass of the asymptotically Euclidian space-time without source ($T_i{}^k = 0$) is positive ($m \geqslant 0$). The weak features of the derivations of these results (assumption that the usual extremum theorems for functions are also valid for functionals, assumption that non-trivial asymptotically Euclidian solutions exist which are non-singular, use of the weak field approximation, use of a particular coordinate atlas) were noted by the authors themselves.

In the preceding paper (Folomeshkin, 1974) I have shown that a class of the asymptotically Euclidian space-times exist for which $m = 0$ in the t-symmetrical case, i.e. when $\partial_0 g_{ik} = 0$. The coordinate atlases of this class of the space-times satisfy a simple additional condition (in the t-orthogonal metric), $\partial_0(\det g_{\alpha\beta}) = 0$. The coordinate atlas of Weyl–Levy-Civita which was used, for example, by Brill (1959) does not satisfy this condition.

The results of Brill (1959) and Folomeshkin (1974) show that the choice of some additional 'coordinate' conditions determine not only the coordinate atlas but the space-time also, i.e. some class of the solutions of the Einstein equations which transform one into another under the isometries of the connected four-dimensional para-compact Hausdorff psuedo-Riemannian manifold with a normal signature $(+ - - -)$.

In the present paper I show that a more general class of the asymptotically Euclidian space-times exist for which the S-mass is equal to zero when $T_i{}^k = 0$.

It is necessary to underline that this result does not prohibit the existence of the different non-isometric (but asymptotically Euclidian also) space-times with a non-zero value of the S-mass when $T_i{}^k = 0$. And this result does not mean that (1) the space-time is flat, (2) the gravitational waves are non-detectable, (3) the energy of the source (detector) of the gravitational waves is not changed in the process of the emission (absorption) of the waves. The mechanical effect of the gravitational waves is determined by the curvature tensor and has no direct relation to the S-mass (energy) of the gravitational waves.

2. *Definition of the S-Mass*

The scalar curvature can be divided into two parts

$$R = G + \partial_k \omega^k/\sqrt{(-g)}, \qquad \text{where } G = g^{ik}(\Gamma^n_{im}\Gamma^m_{kn} - \Gamma^m_{nm}\Gamma^n_{ik})$$

and

$$\omega^k = \sqrt{(-g)}(g^{mn}\Gamma^k_{mn} - g^{kn}\Gamma^m_{nm})$$

In the arbitrary coordinate system the equations $8\pi T_i{}^k = R_i{}^k - \delta_i{}^k R/2$ can be written down in the form

$$8\pi\sqrt{(-g)}(T_i{}^k - \delta_i{}^k T/2) = -\partial_n L_i{}^{kn} + \left(\partial_i \omega^k + \frac{\partial\sqrt{(-g)}G}{\partial g_k^{mn}} g_i^{mn}\right)\Big/ 2$$

where $g_i^{mn} = \partial_i g^{mn}$ and $L_i^{kn} = \sqrt{(-g)}(g^{mn}\Gamma^k_{im} - g^{km}\Gamma^n_{im})/2$. In the static case we have

$$8\pi\sqrt{(-g)}(T_0{}^0 - T/2) = -\partial_\alpha L_0^{0\alpha}, \qquad (\alpha = 1, 2, 3) \tag{2.1}$$

Assuming that as $r \to \infty$ we have an asymptotically Euclidian metric, e.g. in the standard form

$$ds^2 = (1 - 2m/r)\,dt^2 - dr^2/(1 - 2m/r) - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{2.2}$$

we obtain that

$$-\frac{1}{4\pi}\int \partial_\alpha L_0^{0\alpha}\, dV = m \tag{2.3}$$

and therefore

$$m = 2\int (T_0{}^0 - T/2)\sqrt{(-g)}\, dV \tag{2.4}$$

This is a well-known result (Nordström, 1918; Tolman, 1930; Wittaker, 1935; Ruse, 1935; Klark, 1947; Papapetrou, 1947). Equations (2.3) and (2.4) are the generalisations to the non-Euclidian space-time of the usual Gauss theorem (Whittaker, 1935).

We will consider equations (2.2)-(2.4) as the definition of the S-mass of an asymptotically Euclidian system.

For an arbitrary non-static system we have instead of equation (2.1)

$$-\partial_\alpha L_0^{0\alpha} = 8\pi\sqrt{(-g)}(T_0{}^0 - T/2) - \left(\partial_0\omega^0 + \frac{\partial\sqrt{(-g)}G}{\partial g_0^{mn}} g_0^{mn}\right)\Big/ 2$$

and instead of equation (2.4)

$$m = 2\int (T_0{}^0 - T/2)\sqrt{(-g)}\, dV - \frac{1}{8\pi}\left(\partial_0\omega^0 + \frac{\partial\sqrt{(-g)}G}{\partial g_0^{mn}} g_0^{mn}\right) dV \tag{2.5}$$

3. *S-Mass and the Generalised 'Energy-Momentum' Complex*

Equation (2.5) is simply the integral form for one of the Einstein equations in the asymptotically Euclidian case. It is interesting to compare equality (2.5) with the definition of the mass with a generalised complex t_i^k (for example, with the Einstein complex)

$$\mu = \int (T_0^0 + t_0^0)\sqrt{(-g)}\, dV$$
$$t_i^k = \left(G\delta_i^k - \frac{\partial G}{\partial g_k^{mn}} g_i^{mn}\right) \Big/ (16\pi) \tag{3.1}$$

Let us substitute on the right-hand side of equation (2.5) instead of $(T_0^0 - T)$ its expression according to the field equations. We obtain

$$m = \int (T_0^0 + t_0^0)\sqrt{(-g)}\, dV - \frac{1}{8\pi}\int \partial_\alpha (L_0^{0\alpha} - \omega^\alpha/2)\, dV$$

We see that the quantity μ defined in equation (3.1), where t_i^k is the Einstein complex, is not equal to the S-mass of the system and differs from it by the quantity

$$\Delta = m - \mu = -\frac{1}{8\pi}\int \partial_\alpha (L_0^{0\alpha} - \omega^\alpha/2)\, dV$$

If we calculate Δ in different asymptotically Euclidian coordinate systems we will find that the quantity Δ will depend on the coordinate atlas and can have any value. This known 'paradox' means that the quantity μ, defined according to equation (3.1), has no relation to the S-mass of the system in the general case.

4. *S-Mass of an Asymptotically Euclidian System of Gravitational Waves*

The Einstein equation is known to be invariant under arbitrary isometric diffeomorphisms of space-time. Therefore, in the general case, we can impose some additional conditions on the components of the metric tensor. One of the known subsidiary conditions is the harmonic condition (De Donder, 1921; Lanczos, 1923; Fok, 1961), $\partial_i(\sqrt{(-g)}g^{ik}) = 0$. Other possible subsidiary conditions of this type are the following:

$$\partial_i(-gg^{ik}) = 0 \tag{4.1}$$

The S-mass of the t-symmetrical asymptotically Euclidian space-time with $T_i^k = 0$, the coordinate atlas of which satisfies conditions (4.1), is equal to zero (Folomeshkin, 1974). This result is valid also for a more broad class of space-times. The coordinate atlases of these space-times satisfy one subsidiary condition only

$$\partial_k(-gg^{0k}) = 0 \tag{4.2}$$

When considering a more general case of the asymptotically Euclidian space-time without source, it is natural to choose such subsidiary conditions which, for the t-symmetrical moment of time, arrive at conditions (4.2) or (4.1). We choose two subsidiary conditions: condition (4.2) and the condition

$$\Gamma^0_{ik}\,\partial_0 g^{ik} - \Gamma^k_{ik}\,\partial_0 g^{0i} = 0 \tag{4.3}$$

In the t-orthogonal metric

$$ds^2 = g_{00}\,dt^2 - g_{\alpha\beta}\,dx^\alpha\,dx^\beta$$

conditions (4.2) and (4.3) take simple forms:

$$\partial_0(\det g_{\alpha\beta}) = 0 \qquad \text{and} \qquad (\partial_0 g^{\alpha\beta})(\partial_0 g_{\alpha\beta}) = 0$$

Any asymptotically Euclidian coordinate atlas satisfies condition (4.3) in the limit $r \to \infty$ and can easily be transformed to the form which satisfies condition (4.2), i.e. these conditions are consistent with the asymptotically Euclidian character of the space-time.

For the class of coordinate atlases which satisfies conditions (4.2) and (4.3) equation (2.5) takes the simple form (2.4) and we obtain

$$m = 0 \qquad \text{when} \qquad T_i^k = 0 \tag{4.4}$$

Condition (4.3) may be considered as some form of generalisation of the time-symmetry initial condition $\partial_0 g^{ik} = 0$.

5. *Conclusion*

We have shown that a class of the asymptotically Euclidian space-times exist for which $m = 0$ when $T_i^k = 0$. This result does not prohibit the existence of other non-isometric space-times with a non-zero value of the S-mass when $T_i^k = 0$. The specific example of such non-isometric space-time is presented by Brill (1959).

Another simple example, when the solutions of the Einstein equations corresponding to different 'coordinate' conditions correspond, in reality, to different non-isometric space-times, are the known solutions for the static gravitational field of a point particle in the vacuum: the standard Schwarzschild solution (S), harmonic solution (H), isotropic solution (I), the axially symmetric Weyl solution. The S-metric is the isometric extension of the H-metric, and the region $r < m$ of the S-metric is placed beyond the boundaries of the space-time with H-atlas (it corresponds to negative values of the radius in the H-metric). The I-space $R^I_+(r) = \{x \in R_3{}^I \,|\, r \geqslant 0\}$ is mapped (twice) in the proper subset $R^S_{2m}(r) = \{x \in R_3{}^S \,|\, r \geqslant 2m\}$ of the S-space and in the proper subset $R_m{}^H$ of the H-space. The regions $r < 2m$ in the S-space and $r < m$ in the H-space lies beyond the boundaries of the I-space. These space-times are therefore different non-isometric space-times.

This simple example shows that in the general case the additional conditions are not 'coordinate conditions' but space-time conditions.

As we have already noted, the equality of zero of the S-mass of the space-time without source does not mean that the energy of the source (receiver) is not changed in the process of the emission (absorption) of the gravitational waves. Many authors (Bondi, 1957; Bondi, Pirani & Robinson, 1959; Bonnor, 1959; Bondi, van der Burg & Metzner, 1962; Newman & Unti, 1962; Sachs, 1962; Bonnor & Rotenberg, 1961, 1966; Tamburnio & Winicour, 1966; Rotenberg, 1972) have shown, without use of the notions 'energy-momentum pseudo-tensor' or 'energy' of the gravitational waves, that the S-mass of the source (receiver) is changed in the process of the emission (absorption) of the gravitational waves.

Acknowledgement

I thank B. A. Arbuzov, D. R. Brill, S. Deser, L. D. Faddeev and S. S. Gershtein for the discussion of the problems considered in the present paper.

References

Araki, H. (1959). *Annals of Physics*, 7, 456.
Arnowitt, S., Deser, S. and Misner, C. (1960). *Annals of Physics*, 11, 116.
Bauer, H. (1918). *Physikalische Zeitschrift*, **19**, 163.
Bergmann, P. G. and Brunnings, J. H. M. (1948). *Review of Modern Physics*, **21**, 480.
Bergmann, P. G. (1949). *Physical Review*, **75**, 680.
Bergmann, P. G., Penfield, R., Schiller, R. and Zatzkis, H. (1950). *Physical Review*, **80**, 81.
Bergmann, P. G. and Thomson, R. (1953). *Physical Review*, **89**, 400.
Bergmann, P. G. (1956). In *Jubliee of Relativity Theory*. Birkhauser Verlag, Basel.
Bessel-Hagen, E. (1921). *Annals of Mathematics*, **84**, 258.
Bondi, H. (1957). *Nature (London)*, **179**, 1072.
Bondi, H., Pirani, F. and Robinson, I. (1959). *Proceedings of the Royal Society*, **A251**, 519.
Bondi, H., Van der Burg, M. and Metzner, A. (1962). *Proceedings of the Royal Society*, **269**, 21.
Bonnor, W. B. (1959). *Philosophical Transactions of the Royal Society*, **251**, 233.
Bonnor, W. B. and Rotenberg, M. A. (1961). *Proceedings of the Royal Society*, A265, 109.
Bonnor, W. B. and Rotenberg, M. A. (1966). *Proceedings of the Royal Society*, **289**, 247.
Born, M. (1937). *Annales de l'Institut Henri Poincaré*, 7, 155.
Brdicka, M. (1951). *Proceedings of the Royal Irish Academy*, **A54**, 137.
Brill, D. R. (1959). *Annals of Physics*, 7, 466.
Brill, D. R., Deser, S. and Faddeev, L. D. (1968). *Physics Letters*, **A-26**, 538.
Brill, D. R. and Deser, S. (1968). *Physical Review Letters*, **20**, 75.
Cahen, M. and Sengier-Diels, J. (1963). *Bulletin de l'Académie r. de Belgique, Classe des Sciences*, **49**, 43.
Capella, M. A. (1961). *Comptes Rendus Hebdomadaire des Séances de l'Académie des Sciences, Paris*, **252**, 3940.
De Donder, T. (1921). *La Gravifique Einsteinienne*. Paris.
De Wet, J. S. (1947). *Proceedings of the Cambridge Philosophical Society*, **43**, 511.
Eddington, A. S. (1923). *The Mathematical Theory of Relativity*. University Press, Cambridge.
Einstein, A. (1916). *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, **2**, 1111.
Fok, V. A. (1961). *Theory of Space, Time and Gravitation*. Publishing House of Phys.-Math. Literature, Moscow.

Folomeshkin, V. N. (1967). *Yadernaya Fizika*, **6**, 1080.
Folomeshkin, V. N. (1969). *Acta Physica Polonica*, **36**, 439.
Folomeshkin, V. N. (1970). *Acta Physica Polonica*, **B1**, 13.
Folomeshkin, V. N. (1971). Institute of High Energy Physics Preprint, IHEP 71-120, Serpukhov.
Folomeshkin, V. N. (1972). Institute of High Energy Physics Preprint, IHEP 72-92, Serpukhov.
Folomeshkin, V. N. (1974). *International Journal of Theoretical Physics*, Vol. 10, No. 3, p. 145.
Fletcher, J. (1960). *Review of Modern Physics*, **32**, 65.
Goldberg, J. N. (1953). *Physical Review*, **89**, 263.
Goldberg, J. N. (1958). *Physical Review*, **111**, 315.
Havas, P. and Goldberg, J. N. (1962). *Physical Review*, **128**, 398.
Hilbert, D. (1915). *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Klasse*, **3**, 395.
Hilbert, D. (1924). *Annals of Mathematics*, **92**, 1.
Hill, E. L. (1951). *Review of Modern Physics*, **23**, 253.
Hu, N. (1947). *Proceedings of the Royal Irish Academy*, **A51**, 87.
Infeld, L. and Sheidegger, A. E. (1951). *Canadian Journal of Mathematics*, **3**, 195.
Infeld, L. (1953). *Canadian Journal of Mathematics*, **5**, 17.
Infeld, L. (1956). *Annals of Mathematics*, **17**, 545.
Infeld, L. (1959). *Annals of Physics*, **16**, 341.
Infeld, L. and Plebansky, J. (1960). *Motion and Relativity*. Pergamon Press, New York.
Iskraut, R. (1942). *Zeitschrift für Physik*, **119**, 659.
Klark, G. L. (1947). *Proceedings of the Royal Society of Edinburgh*, **62**, 412.
Klein, F. (1917). *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Klasse*, 469.
Klein, F. (1918). *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Klasse*, 171.
Klein, F. (1918). *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Math.-Phys. Klasse*, 394.
Komar, A. (1963). *Physical Review*, **129**, 1873.
Kuchar, K. and Langer, J. (1963). *Czechoslovak Journal of Physics*, **B13**, 233.
Lanczos, K. (1923). *Physikalische Zeitschrift*, **23**, 537.
Langer, J. (1963). *Czechoslovak Journal of Physics*, **B13**, 789.
Lopez, C. A. (1965). *Nuovo Cimento*, **A40**, 19.
Lorentz, H. A. (1915). *Verslagen van de gowone vergadering der wis- en natuurkundige afdeeling. K. Akademie van Wetenschappen te Amsterdam*, **23**, 1073.
Lorentz, H. A. (1916a). *Verslagen van de gowone vergadering der wis- en natuurkundige afdeeling. K. Akademie van Wetenschappen te Amsterdam*, **24**, 1389.
Lorentz, H. A. (1916b). *Verslagen van de gowone vergadering der wis- en natuurkundige afdeeling. K. Akademie van Wetenschappen te Amsterdam*, **24**, 1759.
Lorentz, H. A. (1916c). *Verslagen van de gewone vergadering der wis- en natuurkundige afdeeling. K. Akademie van Wetenschappen te Amsterdam*, **25**, 468.
Lorentz, H. A. (1916d). *Verslagen van de gewone vergadering der wis- en natuurkundige afdeeling. K. Akademie van Wetenschappen te Amsterdam*, **25**, 1380.
Misner, C. W. (1963). *Physical Review*, **130**, 1590.
Mitskevich, N. V. (1958). *Annalen der Physik*, **1**, 319.
Møller, C. (1958a). *Annals of Physics*, **4**, 347.
Møller, C. (1958b). In *Max-Planck Festschrift*, p. 139. Berlin.
Møller, C. (1959). *Kongelige Danske Videnskabernes Selskabs, Matematisk-fysiske (Meddelelser)*, **31**, 14.
Møller, C. (1961). *Annals of Physics*, **12**, 118.
Møller, C. (1964). In *Atti del convegno sulla relativita generale: problemi dell energia e ondre gravitazionalli*, Vol. 2. Roma.
Morgan, T. A. and Joseph, D. W. (1965). *Nuovo Cimento*, **39**, 494.

Newman, E. T. and Unti, T. W. J. (1962). *Journal of Mathematical Physics,* **3**, 891.
Nordstrom, G. (1918). *Proceedings K. Nederlandse Academie van Wetenschappen Amsterdam,* **20**, 1076.
Papapetrou, A. (1947). *Proceedings of the Royal Irish Academy,* **A51**, 191.
Pauli, W. (1921). Relativitatstheorie. *Encyklopadie der Mathematik Wissenschaften,* **2**, 539.
Pauli, W. (1941). *Review of Modern Physics,* **13**, 203.
Penrose, R. (1968). *Structure of Space-Time.* W. A. Benjamin, Inc., New York.
Peres, A. (1959). *Nuovo Cimento,* **11**, 644.
Petrov, A. Z., Piragas, K. A. and Dobrovolsky, V. A. (1968). In *Gravitation and Relativity,* No. 4-5, p. 44. University Publishing House, Kazan.
Pirani, F. A. E. (1961). In *Gravitation: Introduction to Current Research.* Wiley, New York.
Plybon, B. (1971). *Journal of Mathematical Physics,* **12**, 57.
Ray, J. R. (1968). *Nuovo Cimento,* **A56**, 189.
Rosen, N. (1956). In *Jubilee of Relativity Theory,* p. 170. Birkhauser Verlag, Basel.
Rotenberg, M. A. (1972). *Journal of Physics A,* **5**, 198.
Scahs, R. K. (1962). *Proceedings of the Royal Society,* **270**, 102.
Schrödinger, E. (1918). *Physikalische Zeitschrift,* **19**, 4.
Schrödinger, E. (1948). *Proceedings of the Royal Irish Academy,* **A52**, 1.
Schrödinger, E. (1951). *Proceedings of the Royal Irish Academy,* **A54**, 79.
Sexl, R. U. (1966). *Physics Letters,* **20**, 376.
Sheidegger, A. E. (1951). *Physical Review,* **82**, 833.
Scheidegger, A. E. (1953). *Review of Modern Physics,* **25**, 451.
Sheidegger, A. E. (1955). *Physical Review,* **99**, 1883.
Shirokov, M. F. (1972a). In *Gravitation.* Publishing House 'Naukova Dumka', Kiev.
Shirokov, M. F. (1972b). Institute of Theoretical Physics Preprint, ITP-72-131E, Kiev.
Tamburino, L. A. and Winicour, J. H. (1966). *Physical Review,* **150**, 1039.
Tolman, R. C. (1930). *Physical Review,* **35**, 875.
Tolman, R. C. (1934). *Relativity, Thermodynamics and Cosmology.* Clarendon Press, Oxford.
Trautman, A. (1956). *Bulletin de l'Académie Polonaise des Sciences,* **4**, 665.
Trautman, A. (1958). *Bulletin de l'Académie Polonaise des Sciences,* **6**, 627.
Tratuman, A. (1958b). In *Lectures on General Relativity.* King's College, London.
Trautman, A. (1964). In *Lectures on General Relativity.* Prentice-Hall, New Jersey.
Weber, J. and Wheeler, J. (1957). *Review of Modern Physics,* **29**, 509.
Weyl, H. (1917). *Annalen der Physik,* **54**, 117.
Weyl, H. (1918). *Raum-Zeit-Materie.* Springer Verlag, Berlin.
Whittaker, E. T. (1935). *Proceedings of the Royal Society,* **149**, 384.
Zatzkis, H. (1951). *Physical Review,* **81**, 1023.